Maximum Likelihood Parallel Decoding Metrics for Quasi-Orthogonal Space-Time Block Codes: Derivation and Lowpower Implementation

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Abstract Orthogonal Space-Time Block Codes (STBCs), of which the Alamouti Code is the best known, allow parallel maximum-likelihood (ML) decoding and full diversity. Yet rate-one, orthogonal designs for complex-valued constellations only exist for two-antenna transmitters. Quasi-Orthogonal Space-Time Block Codes overcome this limitation. These codes allow more than two transmit antennas, provide rate-one codes, and can produce full diversity. QOSTBCs are therefore suitable for multi-antenna base-stations serving single-antenna mobile terminals. This paper describes the derivation and low-power digital implementation of parallel metrics for ML decoding of QOSTBCs.

Key words STBC, Maximum-Likelihood, Decoding, Power Consumption

1 Introduction

The objective of this paper is to derive an energy-efficient maximum likelihood (ML) decoding metric for a particular Quasi-Orthogonal Space-Time Block Code (QOSTBC) [1]. For this purpose, a QOSTBC code with four transmit antennas is considered. The generator matrix **G** of the code is:

$$\mathbf{G} = \begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \\ 0 & 0 & x3 & x4 \\ 0 & 0 & -x_4^* & x_3^* \end{bmatrix} \tag{1}$$

Where x_n is the indeterminate variable. All entries are complex numbers. The codeword matrix **C** for **G** is obtained through the following replacements:

$$\begin{aligned} x_1 &= s_1 + s_2 \\ x_2 &= s_3 + s_4 \\ x_3 &= s_1 - s_2 \\ x_4 &= s_3 - s_4 \end{aligned} \tag{2}$$

This substitution allows every symbol s to be transmitted at every transmission time. So the code is a full-rate code, or has a rate of one. Furthermore, the code provides full diversity for transmitted symbols, as each symbol can enjoy the path provided by each antenna. In effect, during each encoding instant, four symbols $\{x_n, n = 1, 2, 3, 4\}$ are emitted by the transmitter. After going through a channel **H**, and experiencing corruption by noise **W**, the symbols form each antenna m can be written as:

$$\mathbf{Y} = \mathbf{HX} + \mathbf{W} \tag{3}$$

To perform the decoding, we need four received symbols from each antenna. The received symbols $\mathbf{y_m}$ from each antenna m necessary for decoding are then arranged as:

$$\mathbf{y}_m = \begin{bmatrix} y_{1,m} & y_{2,m} & y_{3,m} & y_{4,m} \end{bmatrix} \tag{4}$$

So the received signal is the matrix:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_M \end{bmatrix} \tag{5}$$

Where have assumed that we have a total of M antennas. In this paper we focus on a 4 × 2 configuration, so we have two receive antennas. This is particularly suitable for mobile terminals where space is limited and the use of two receive antennas for diversity purposes is common practice. We also need channel estimates, i.e. we need an estimate the matrix **H**. Let us assume perfect channel estimates, and similar to **Y**, define H as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_M \end{bmatrix} \tag{6}$$

With the definitions above, a decoding scheme for the QOSTBC defined by Equation 1 is described.

2 Maximum-Likelihood Decoding of QOSTBC

In [2] it is shown that the maximum-likelihood (ML) decoding of QOSTBC amounts to the following minimization problem:

$$\min_{s_1,s_2,s_3,s_4} \{\mathbf{H}^H\mathbf{C}^H\mathbf{CH} - \mathbf{H}^H\mathbf{C}^H\mathbf{y} - \mathbf{y}^H\mathbf{CH}\} \tag{7}$$

Manipulation of Equation 7 shows that the ML decoding algorithm reduces to the minimization of two terms. The first is:

$$(\tilde{s}_1, \tilde{s}_2) = \arg\min_{s_1,s_2} \psi_{12}(s_1, s_2) \tag{8}$$

and the second is:

$$(\tilde{s}_3, \tilde{s}_4) = \arg\min_{s_3,s_4} \psi_{34}(s_3, s_4) \tag{9}$$

We spare the reader the details of algebraic manipulation and give the following results:

$$\psi_{12} = \sum_{m=1}^{M} S_{12}A_m + 2\mathrm{Re}\{D_m s_1 + D'_m s_2\} + S'_{12}B_m \tag{10}$$

and:

$$\psi_{34} = \sum_{m=1}^{M} S_{34}A_m + 2\mathrm{Re}\{E_m s_3 + E'_m s_4\} + S'_{34}B_m \tag{11}$$

The parameters of Equations 8 and 9 are listed in Tables 1 and 2. Note that the functions ψ_{12} and ψ_{34} are completely independent and can be computed in parallel. They also share the parameters A_m and B_m.

Table 1: Parameters of $\psi_{12}(s_1, s_2)$

$S_{12} = |s_1|^2 + |s_2|^2$
$A_m = \sum_{n=1}^{4} |h_{n,m}|^2$
$D_m = -h^*_{m,n}y_{1,m} - h_{2,m}y^*_{2,m} - h^*_{3,m}y_{3,m} - h_{4,m}y^*_{4,m}$
$D'_m = -h^*_{m,n}y_{1,m} - h_{2,m}y^*_{2,m} + h^*_{3,m}y_{3,m} + h_{4,m}y^*_{4,m}$
$B_m = |h_{1,m}|^2 + |h_{2,m}|^2 - |h_{3,m}|^2 - |h_{4,m}|^2$
$S'_{12} = \mathrm{Re}\{s_1 s^*_2\}$

Table 2: Parameters of $\psi_{34}(s_3, s_4)$

$S_{34} = |s_3|^2 + |s_4|^2$
$E_m = h_{m,n}y^*_{1,m} - h^*_{2,m}y_{2,m} + h_{3,m}y^*_{3,m} - h^*_{4,m}y_{4,m}$
$E'_m = h_{m,n}y^*_{1,m} - h^*_{2,m}y_{2,m} - h_{3,m}y^*_{3,m} + h^*_{4,m}y_{4,m}$
$S'_{34} = \mathrm{Re}\{s_3 s^*_4\}$

3 Lowpower Implementation of QOSTBC Decoder

Tables 1 and 2 indicate that nearly half of decoding complexity goes to the computations of A_m and B_m. The D_m and E_m terms, as well as their primes, make up the other half. The S parameters are lookup tables. Thus, in order to reduce the power consumption of the decoder, the focus must be on approximations for A_m and B_m. Before this, we must introduce a useful metric.

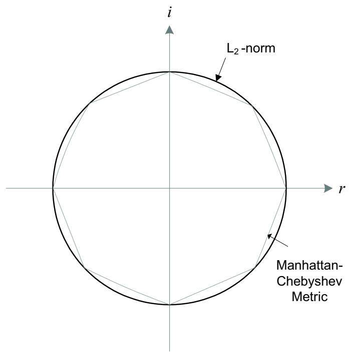

Figure 1: L_2-norm and the Manhattan-Chebyshev Metric $\mu(p, q)$

3.1 The Manhattan-Chebyshev Metric

In citeRohani, it was shown that the absolute value of a complex number $p + jq$ can be accurately approximated by an average of its L_1 and its L_∞ norms. In other words:

$$L_2(p, q) \approx \frac{1}{2}\left(L_1(p, q) + L_\infty(p, q)\right) \tag{12}$$

where $L_2(p, q) = (p^2 + q^2)^{1/2}$, $L_1(p, q) = |p - q|$, and $L_\infty(p, q) = \max(|p|, |q|)$.

To manage the notation, it is convenient to let $\mu(p, q)$ be:

$$\mu(p, q) = \frac{1}{2}\left(L_1(p, q) + L_\infty(p, q)\right) \tag{13}$$

We call this the Manhattan-Chebyshev Metric (MCM) because L_1 is known as the Manhattan norm, and L_∞ is known as the Chebyshev norm. (Note that μ shall be used frequently in what follows and its representation shall be adjusted for clarity). Figure 1 shows the contours of $L_2(p, q)$ and $\mu(p, q)$ and gives a sense of the accuracy of MCM. The motivation here is not hard to surmise. Both A_m and B_m terms are made of sums of squared L_2 norms of the channel estimates $h_{n,m}$. By approximating the L_2 norm, substantial savings shall be possible.

3.2 Power Consumption Estimates

Simple logic circuits for implementation of $L_2^2(p, q)$ and $\mu(p, q)$ are shown in Fig. 2. We synthesized these circuits for $L_2^2(p, q)$ and $\mu(p, q)$ in a 90nm TSMC 6-metal process and the power consumption estimates for the arithmetic building blocks are shown in Table 3.

These results indicate power consumption of $\mu(p, q)$ is almost 10% of $L_2^2(p, q)$. Note that $\mu(p, q)$ approximates L_2, and not L_2^2. The latter is of course used in both A_m and B_m. However, due to their relatively larger power consumption, multiplications should be eliminated as much as possible. Fortunately, this turns out not to be a major issue, as shown below.

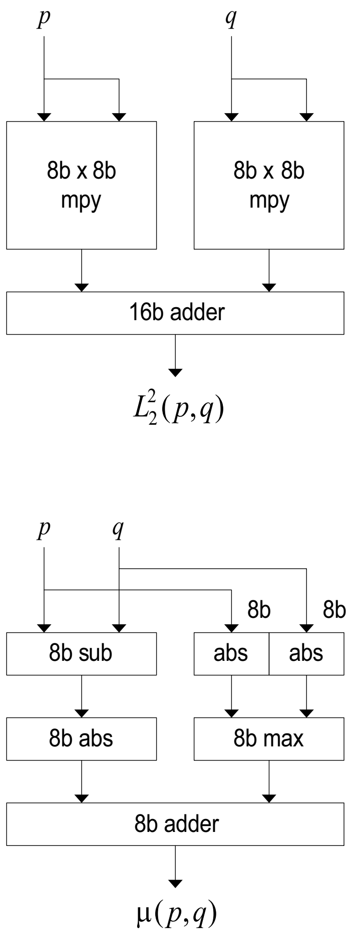

Figure 2: Circuits for calculation of $L_2^2(p,q)$ and $\mu(p,q)$

3.3 Approximating B_m and A_m

First, let us consider B_m and re-write it as:

$$B_m = \left(|h_{1,m}|^2 + |h_{2,m}|^2\right) - \left(|h_{3,m}|^2 + |h_{4,m}|^2\right) \quad (14)$$

We can approximate this as:

$$\tilde{B}_m = \left(\mu^2\left(|h_{1,m}|, |h_{2,m}|\right)\right) - \left(\mu^2\left(|h_{3,m}|, |h_{4,m}|\right)\right) \quad (15)$$

Effectively:

$$\tilde{B}_m = \mu_{12}^2 - \mu_{34}^2 = (\mu_{12} - \mu_{34}) \times (\mu_{12} + \mu_{34}) \quad (16)$$

here the subscripts of μ terms relate to those of the channel estimates. Equation 16 shows that only one multiplication can be used to calculate $\tilde{B}_m$. Figure 3 provides a visual aid for clarification of how μ_{12} and μ_{34} can be derived from $h_{n,m}$ through repeated application of MCM.

To complete the calculation of $\tilde{B}_m$, the four terms corresponding to $h_{n,m}$ must also be computed. These are in turn approximated by:

$$|h_{n,m}| = \mu_n\left(\mathrm{Re}\{h_{n,m}\}, \mathrm{Im}\{h_{n,m}\}\right) \quad (17)$$

Table 3: Power Consumption of Arithmetic Building Blocks

Block	Size	Power μW
mpy	8b	28.2
add/sub	8b	1.8
	16b	4.5
max	8b	1.5
abs	8b	0.6

Table 4: Power Consumption of $L_2^2(p,q)$ and $\mu(p,q)$

Block	$L_2^2(p,q)$	$\mu(p,q)$
mpy	2 x 28.2	0
add/sub	4.5	2 x 1.8
max	0	1.5
abs	0	3 x 0.6
Total	60.9 μW	6.9 μW

For approximation of A_m, we have the $|h_{n,m}|$ available from Equation 17 and these could be squared by 8-bit multipliers and then summed. However, an inequality due to [4] was found effective for reducing the number of multiplications from four down to only one. The inequality states that for a set of positive numbers $p_1, p_2, \ldots, p_r$, we have:

$$\sum_{i=1}^{r} p_i^2 \geq \frac{1}{r}\left(\sum_{i=1}^{r} p_i\right)^2 \quad (18)$$

The use of this inequality in A_m leads to the following approximation:

$$\tilde{A}_m = \frac{1}{4}\left(\sum_{n=1}^{4} \mu_n\left(\mathrm{Re}\{h_{n,m}\}, \mathrm{Im}\{h_{n,m}\}\right)\right)^2 \quad (19)$$

Note that here we have $r = 4$ and term $1/r$ simply amounts to ignoring the two least significant bits.

4 Simulation Results

The MCM metric was included in a computer simulation to investigate the effect of approximation in the bit error rate (BER) performance of QOSTBC with 16-QAM over a Raleigh fading channel. We assumed perfect channel estimation. Two versions of QOSTBC were simulated: one with no approximations, and another with approximation that we refer to as M-QOSTBC. For reference, we also include BER results for an Orthogonal STBC, whose BER performance is labeled by OSTBC. This is a half-rate STBC for a 4x2 configuration from [5]. We see that OSTBC outperforms QOSTBC by approximately 2 dB. This is to be expected from a half-rate code. What is interesting is the performance of M-QOSTBC. We see some loss in performance which amounts to 0.2 dB to 0.3 dB.

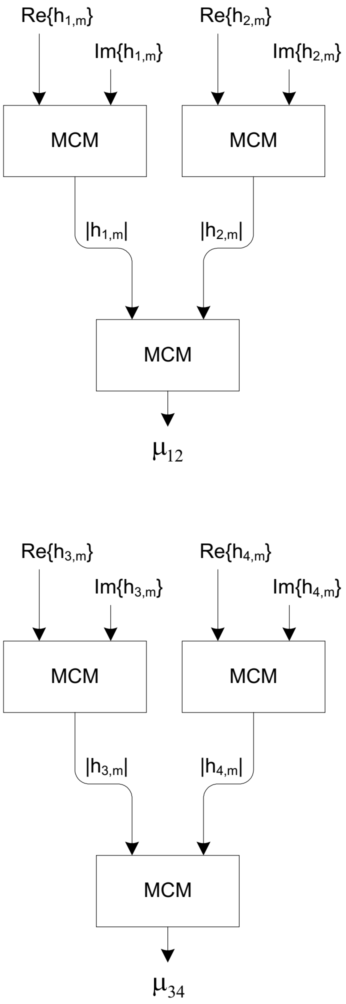

Figure 3: Circuits for Calculation of μ_{12} and μ_{34}

5 Conclusions

By modifying the ML metric of a QOSTBC to make use of MCM, the power consumption of the decoder can be reduced at the expense of 0.2 to 0.3 dB loss in BER performance. This seems a reasonable trade-off for applications that can tolerate this loss yet benefit from lower power consumption.

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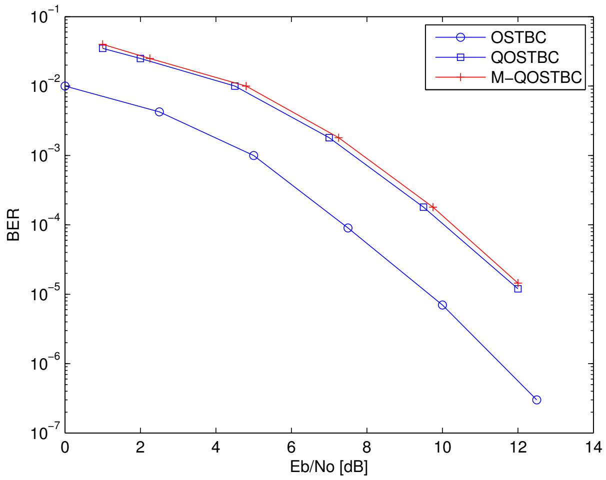

Figure 4: BER Performance of M-QOSTBC compared with QOSTBC and OSTBC