Manhattan-Chebychev Distance Metric for MIMO Systems

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Abstract This paper presents a new distance metric for MIMO detection. Our proposed metric is a combination of Manhattan and Chebychev distances and its digital logic circuitry is simpler and has lower power consumption than the Euclidean Distance (ED). Our simulation results show that the new M-C metric can be used in MIMO detectors, such as the QRM-MLD and B-Chase, instead of the Euclidean Distance (ED) with almost no loss in BER performance.

Key words MIMO Detection, Distance Metrics, QRM-MLD.

1. Introduction

The Euclidean distance (ED) or squared ED are commonly used in communication signal processing algorithms as a measure of the similarity between a received signal **y** and a desired signal **x**. The received signal is a distorted and noisy version of the desired signal **x**.

The Euclidean distance L_2 between ***x*** and ***y*** is defined as follows,

$$\begin{aligned} L_2 &= \|\mathbf{y} - \mathbf{x}\| \\ &= \left[|y_I - x_I|^2 + |y_Q - x_Q|^2 \right]^{\frac{1}{2}} \end{aligned} \qquad (1)$$

Here, x_I, y_I, x_Q, and y_Q represent the in-phase and quadrature components of **x** and **y**, respectively. Also, $|arg|$ represents the absolute value of *arg*. The distance L_2 is also known as the L_2-norm.

Computation of ED as outlined by (1) is costly because of the squared terms and the square-root term in the formula. In some computationally extensive algorithms, the Manhattan distance metric has been used to replace ED and squared ED in order to trade off computational efficiency with performance [1-3]. The Manhattan distance (MD), also known as the city block distance, is given by

$$L_1 = |y_I - x_I| + |y_Q - x_Q| \qquad (2)$$

The Manhattan distance is also known as the L_1-norm.

A comparison between (1) and (2) reveals why the Manhattan distance is preferable to the ED in terms of computational complexity. A yet more efficient metric is the Chebychev or chessboard [4] (CD) distance. This is calculated according to

$$\begin{aligned} L_\infty &= \lim_{p \to \infty} \left[(y_I - x_I)^p + (y_Q - x_Q)^p \right]^{\frac{1}{p}} \\ &= \max\left(|y_I - x_I|, |y_Q - x_Q| \right) \end{aligned} \qquad (3)$$

Here, $\max(arg_1, arg_2)$ is a function whose value is equal to the larger of its arguments arg_1 and arg_2. The Chebychev distance is also known as the L_∞-norm.

Let $a = |y_I - x_I|$ and $b = |y_Q - x_Q|$ represent a point P(X=a, Y=b) on the Cartesian coordinate system X-Y. Then, the loci of all points P with a constant distance r from the origin O, for the Manhattan, Chebychev, and the Euclidian distances are shown in

Figure 1.

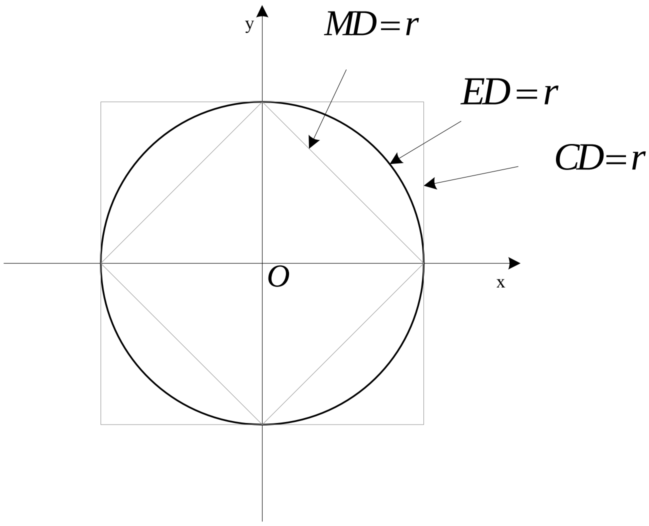

Figure 1 The loci of point P with a fixed distance r from the origin. The distance metric used determines the geometry of the locus in each case.

It is seen from Figure 1 that points on $L_1=r$ are closer to the origin O than points on $L_2=r$ even though both are considered to have the same norms, i.e. $L_1=L_2=r$. Therefore, using the Manhattan distance L_1 as an approximation of the Euclidian distance L_2 results in an overestimation of the distance. Similarly, using the Chebychev distance L_∞ to approximate the Euclidian distance leads to an underestimated distance.

In what follows we introduce a new distance metric, namely the Manhattan-Chebyshev distance (MCD). We show that its simplicity is comparable with the MD and CD metrics while its accuracy is significantly improved over those of the MD and CD metrics.

2. The Manhattan-Chebychev Distance

The MC distance, $L_{1,\infty}$, between $\boldsymbol{x}$ and $\boldsymbol{y}$ is calculated as the average of the L_1 and L_∞ distances between these signals as follows,

$$\begin{aligned} L_{1\infty} &= \frac{1}{2}\left(L_1 + L_\infty\right) \\ &= \frac{1}{2}\left[\left(\left|y_I - x_I\right| + \left|y_Q - x_Q\right|\right) + \max\left(\left|y_I - x_I\right|, \left|y_Q - x_Q\right|\right)\right] \end{aligned} \quad (4)$$

The locus of points P with $L_{1,\infty}=r$ is shown in Figure 2.

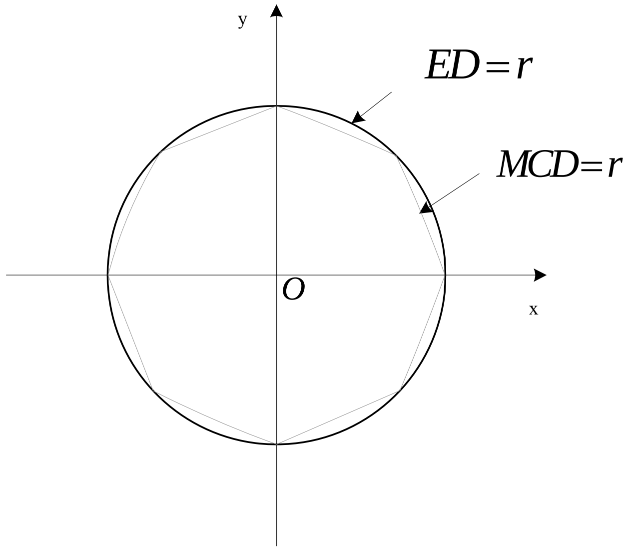

Figure 2 The locus of points with $L_{1,\infty}=r$.

As it can be seen from Figure 2, the locus of $L_{1,\infty}=r$ is an octagon which approximates the circle $L_2=r$ much more closely than the Manhattan or Chebychev distances do individually. We have shown in Figure 3 the relative approximation error of the MC distance in comparison with the Manhattan and the Chebychev errors as a function of the x-ordinate (c.f. Figures 1 and 2). We calculate the relative approximation errors as follows

$$\varepsilon = \left|\frac{L_n - L_2}{L_2}\right| \quad (5)$$

Here, L_n has been used to represent L_1, L_∞, or $L_{1,\infty}$.

It can be observed from Figure 3 that the Manhattan and the Chebychev metrics approximations result in maximum relative errors of ~0.4 and ~0.3, respectively. On the other hand, the use of MCD metric leads to a maximum relative error of only ~0.1.

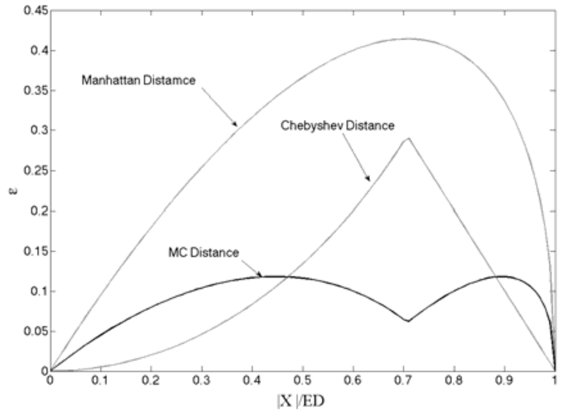

Figure 3 The relative approximation errors of Manhattan, Chebychev, and MCD metrics.

Table 1 Summary of the main simulation parameters

Simulation Parameters	
Channel coding†	8-sates, rate-1/3 Turbo endocing
Data modulation	16QAM
OFDM	1024-point IFFT (768 modulated)
symbol_period	9.259 μsec (including 1.674 μsec guard interval)
MIMO configuration	4×4
Channel model	6-path exponentially decaying Rayleigh fading plus AWGN
	Path gains : [0, -2, -4, -6, -8, -10] dB
	Path delays : [0, 3, 6, 9, 12, 14] *symbol_period
Channel estimates	Perfect
Channel decoder†	Max-log-MAP; 8 iterations

† Only used with the QRM-MLD algorithm

3. Simulation Results

An example of a usage scenario for the MCD metric is in the detection algorithm of a multiple-input multiple-output receiver. In this case, replacing the squared Euclidian distance with the MCD leads to significant reductions in computational complexity. In addition, the performance loss due to the MCD metric is marginal. We have applied the MC metric in two well-known MIMO detection algorithms; the QRM-MLD [5,6], and the B-Chase algorithms. The parameters of our computer simulations of the algorithm are summarized in Table 1. The bit error rate (BER) performances of the algorithm once when using the squared ED and once while using the MCD are shown in Figures 4 and 5.

In Figure 4 We have shown the performances of QRM-MLD before and after soft-decision decoding to show that the MCD performs equally well with and without forward error correcting (FEC) codes.

Figure 5 shows that MCD performs similarly well with the B-Chase MIMO detection algorithm. In this case, only the uncoded performance was simulated because the soft-decision version of the algorithm is not available.

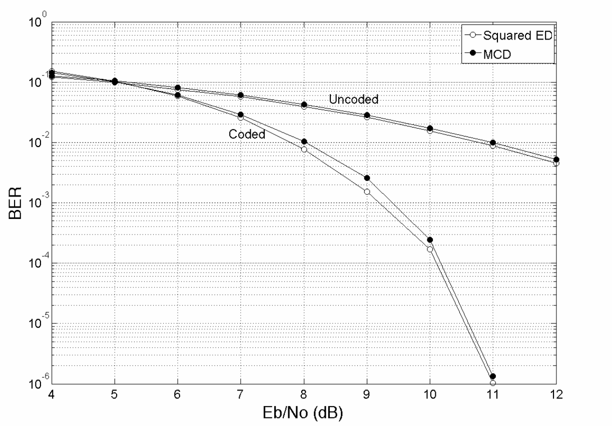

Figure 4 The performance of QRM-MLD algorithm using the MCD in comparison with the squared ED for coded and uncoded transmissions.

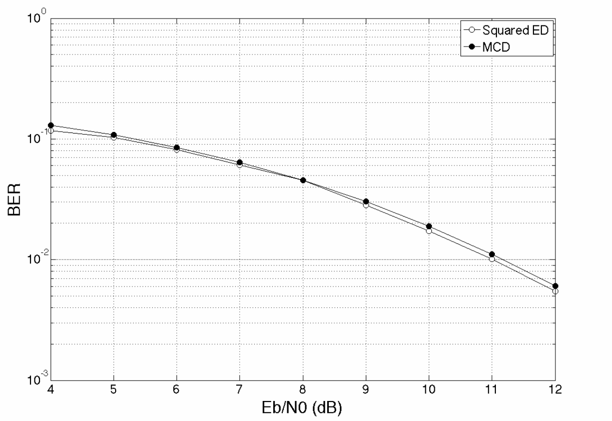

Figure 5 The performance of B-Chase MIMO detection algorithm using the MCD in comparison with the squared ED for an uncoded transmission.

4. Conclusions

We have presented the MCD as a new distance metric for replacing the Euclidean distance in communication signal processing algorithms in general and MIMO detection algorithms in particular. We gave shown that MCD approximates the ED with a maximum error of ~10%. In addition, the MCD requires significantly less computations than the ED as it eliminates multiplication. These features make MCD an attractive alternative to ED for small, low-power implementation of multiple-antenna detectors. It would lead to reduced power consumption and chip area with little loss in performance.

5. References

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